Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel Level 3 GCE

Monday 24 June 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/4C**

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A particle P moves along a straight line.
Initially P is at rest at the point O on the line.

At time t seconds, where $t \geqslant 0$

- the displacement of P from O is x metres
- the velocity of P is $v\,\text{m s}^{-1}$ in the positive x direction
- the acceleration of P is $\dfrac{96}{(3t+5)^3}\,\text{m s}^{-2}$ in the positive x direction

(a) Show that, at time t seconds, $v = p - \dfrac{q}{(3t+5)^2}$, where p and q are constants to be determined.

(4)

(b) Find the limiting value of v as t increases.

(1)

(c) Find the value of x when $t = 2$

(4)

a) $a = \dfrac{dv}{dt}$ so $\dfrac{96}{(3t+5)^3} = \dfrac{dv}{dt}$

Integration by variable separation

$$\int \frac{96}{(3t+5)^3}\,dt = \int 1\,dv$$

$$-\frac{96}{2\times 3\times(3t+5)^2} + C = v$$

CONSTANT

Boundary conditions, at $t=0$, $v=0$

$0 = -\dfrac{96}{150} + C$ $\therefore C = \dfrac{16}{25}$

$\therefore v = \dfrac{16}{25} - \dfrac{96}{6(3t+5)^2}$

simplify

$$v = \frac{16}{25} - \frac{16}{(3t+5)^2}$$

b) as $t \to \infty$, $\dfrac{-16}{(3t+5)^2} \to 0$

denominator 'becomes' infinity

$\therefore$ $v \to \dfrac{16}{25}$ $(= 0.64)$

Question 1 continued

c) $v = \frac{dx}{dt}$ so $\frac{16}{25} - \frac{16}{(3t+5)^2} = \frac{dx}{dt}$

$$\int \frac{16}{25} - \frac{16}{(3t+5)^2}\, dt = \int 1\, dx$$

Integration by variable separation

$$\frac{16}{25}t + \frac{16}{3(3t+5)} + D = x$$

CONSTANT

↳ no need to determine this

"Find x when $t = 2$"

DEFINITE INTEGRAL

$$x = \left[\frac{16}{25}t + \frac{16}{3(3t+5)}\right]_0^2$$

$$x = \left[\frac{32}{25} + \frac{16}{3(11)}\right] - \left[\frac{16}{3(5)}\right]$$

$$x = \frac{192}{275} \quad (= 0.698\ldots)$$

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

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2.

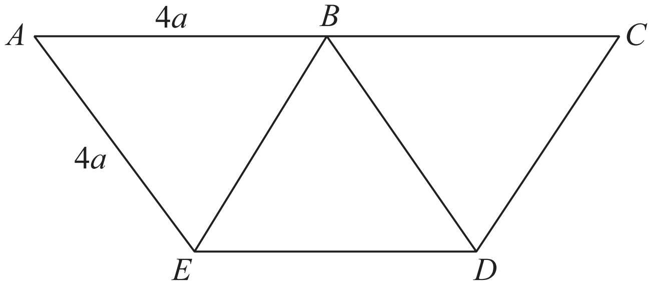


Figure 1

A uniform rod of length $28a$ is cut into seven identical rods each of length $4a$. These rods are joined together to form the rigid **framework** $ABCDEA$ shown in Figure 1.

All seven rods lie in the same plane.

The distance of the centre of mass of the framework from ED is d.

(a) Show that $d = \frac{8\sqrt{3}}{7}a$

(4)

The weight of the framework is W.

The framework is freely pivoted about a horizontal axis through C.

The framework is held in equilibrium in a vertical plane, with AC vertical and A below C, by a **horizontal force** that is applied to the framework at A.

The force acts in the same vertical plane as the framework and has magnitude F.

(b) Find F in terms of W.

(3)

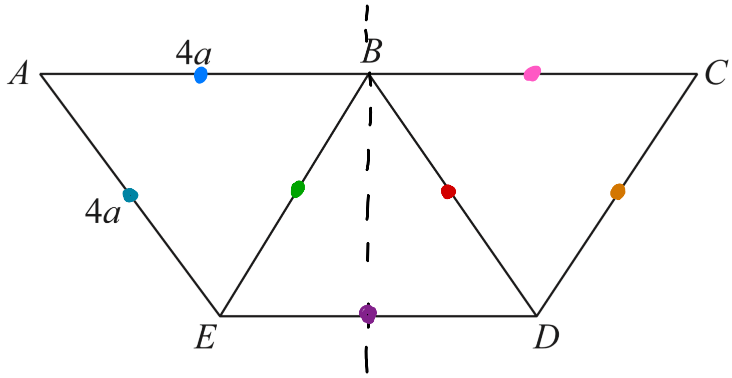
Question 2 continued

a) Since this is a framework, we use moment = force x perpendicular distance

force is proportional to the lengths as they are uniform rods so we can replace force with length since we use

$$\sum m_i x_i = \bar{x} \sum m_i$$

where sum of moments of each component is equal to the singular moment through the COM

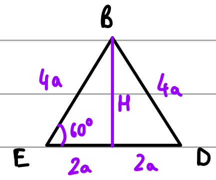


USEFUL TIP!

COM lies on line of symmetry

Notice triangles are EQUILATERAL so angles are 60°

Taking moments about ED



$H = 4a \sin 60$

$28ad = 4a \times 2a\sin 60 + 4a \times 2a\sin 60 + 4a \times 2a\sin 60 +$
$4a \times 2a\sin 60 + 4a \times 4a\sin 60 + 4a \times 4a\sin 60 +$
$4a \times 0$

$= 4 \times 4a \times 2a\sin 60 + 2 \times 4a \times 4a\sin 60$

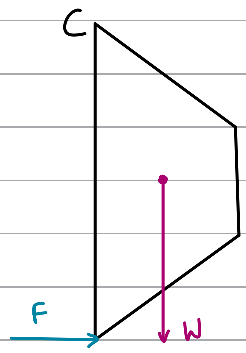
$28ad = 16a^2\sqrt{3} + 16a^2\sqrt{3}$

$28d = 32a\sqrt{3}$

$d = \frac{32\sqrt{3}}{28}a$

$d = \frac{8\sqrt{3}}{7}a$

b)



Taking moments about C

$8a \times F = \left(4a\sin 60 - \frac{8\sqrt{3}}{7}a\right) \times W$

$8aF = \left(2a\sqrt{3} - \frac{8\sqrt{3}}{7}a\right) W$

$8aF = \frac{6\sqrt{3}}{7} a W$

$F = \frac{3\sqrt{3}}{28} W$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

3.

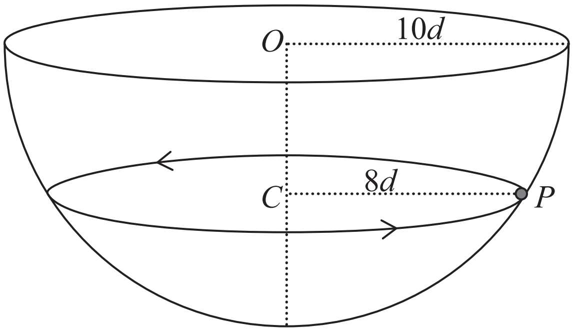


Figure 2

Figure 2 shows a hemispherical bowl of internal radius 10d that is fixed with its circular rim horizontal.

The centre of the circular rim is at the point O.

A particle P moves with **constant** angular speed on the smooth inner surface of the bowl.

The particle P moves in a horizontal circle with radius 8d and centre C.

(a) Find, in terms of g, the exact magnitude of the acceleration of P.

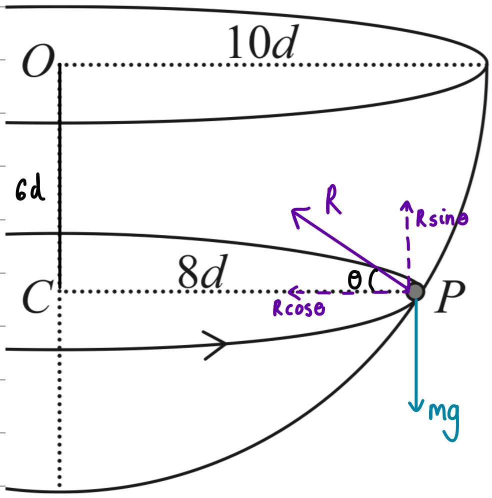
(6)

The time for P to complete one revolution is T.

(b) Find T in terms of d and g.

(3)

a)



Resolving vertically

$R\sin\theta = mg$ ①

Resolving horizontally

$R\cos\theta = ma \; (= mr\omega^2)$ ②

horizontal equation of motion

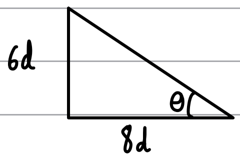
Solving for a:

$\frac{①}{②} = \frac{mg}{ma} = \frac{R\sin\theta}{R\cos\theta}$

$\frac{g}{a} = \tan\theta$

$a = \frac{g}{\tan\theta}$

$a = \frac{4}{3}g$



$\tan\theta = \frac{6d}{8d} = \frac{3}{4}$

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Question 3 continued

b) $a = \omega^2 r$ Equation for centripetal acceleration for circular motion

formulae memorised

$\therefore \omega^2 r = \frac{4}{3} g$

$8d\omega^2 = \frac{4}{3} g$

$\omega = \sqrt{\frac{g}{6d}}$

ALSO $T = \frac{2\pi}{\omega}$ where T = one time period

ω = angular speed

formulae memorised

so $T = \frac{2\pi}{\sqrt{\frac{g}{6d}}}$

$\therefore T = 2\pi \sqrt{\frac{6d}{g}}$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

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4.

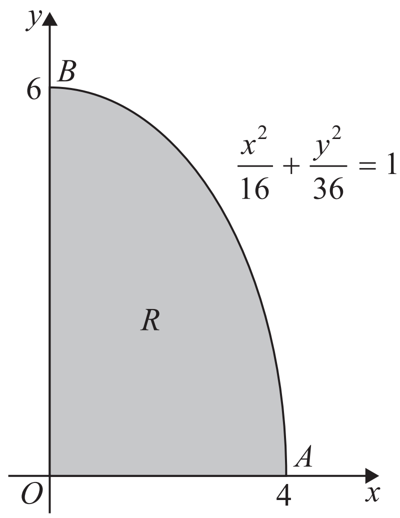


Figure 3

A uniform lamina OAB is in the shape of the region R.
Region R lies in the first quadrant and is bounded by the curve with equation

$\frac{x^2}{16} + \frac{y^2}{36} = 1$, the x-axis, and the y-axis, as shown shaded in Figure 3.

The point A is the point of intersection of the curve and the x-axis.
The point B is the point of intersection of the curve and the y-axis.

One unit on each axis represents 1 m.

The area of R is 6π

The centre of mass of R lies at the point with coordinates $(\bar{x}, \bar{y})$

(a) Use algebraic integration to show that $\bar{x} = \frac{16}{3\pi}$

(5)

(b) Use algebraic integration to find the exact value of $\bar{y}$

(4)

The lamina is freely suspended from A and hangs in equilibrium with OA at angle $\theta°$ to the downward vertical.

(c) Find the value of θ

(3)

a) Re-arrange $\frac{x^2}{16} + \frac{y^2}{36} = 1$ to get y

$$y = \sqrt{36 - \frac{9x^2}{4}}$$

Question 4 continued

A: $(4,0)$

B: $(0,6)$

$$M\bar{x} = \int_a^b \rho x y \, dx$$

To find COM of lamina under y between a and b in x axis

where ρ = mass per unit area (cancels out anyways)

where $M = A \times \rho$

where A = area

ρ = density

Applying formulae

$$6\pi \times \cancel{\rho}\, \bar{x} = \cancel{\rho} \int_0^4 x \sqrt{36 - \frac{9x^2}{4}} \, dx$$

Integration by trial function

Try $y = \left(36 - \frac{9x^2}{4}\right)^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3}{2}\left(36 - \frac{9x^2}{4}\right)^{\frac{1}{2}} \times -\frac{18x}{4}$$

$$= -\frac{27}{4}\left(36 - \frac{9x^2}{4}\right)^{\frac{1}{2}}$$

SCALE : $-\frac{4}{27}$

$$6\pi\bar{x} = \left[-\frac{4}{27}\left(36 - \frac{9x^2}{4}\right)\right]_0^4$$

$$= -\frac{4}{27}(36-36)^{\frac{3}{2}} - \left(-\frac{4}{27}(36)^{\frac{3}{2}}\right)$$

$$6\pi\bar{x} = 32$$

$$\bar{x} = \frac{32}{6\pi}$$

$$\therefore \bar{x} = \frac{16}{3\pi}$$

Question 4 continued

b) $M\bar{y} = \int_a^b \frac{1}{2}\rho y^2 \, dx$

To find COM of lamina under y between a and b in y axis

where ρ = mass per unit area (cancels out anyways)

Applying formulae

$$6\pi \times \cancel{\rho}\, \bar{y} = \frac{1}{2}\cancel{\rho} \int_0^4 36 - \frac{9x^2}{4} \, dx$$

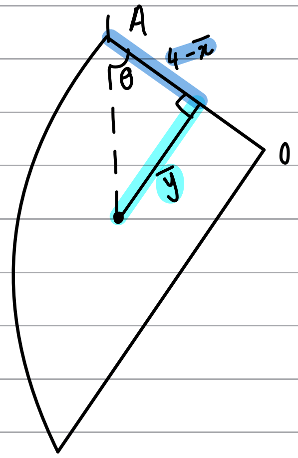
$$6\pi \bar{y} = \frac{1}{2}\left[36x - \frac{3x^3}{4}\right]_0^4$$

$$= \frac{1}{2}(144 - 48)$$

$$6\pi \bar{y} = 48$$

$$\bar{y} = \frac{8}{\pi}$$

c)



$$\tan\theta = \frac{\bar{y}}{4-\bar{x}}$$

$$= \frac{\frac{8}{\pi}}{4 - \frac{16}{3\pi}} = \frac{6}{3\pi - 4}$$

$$\theta = \arctan\left(\frac{6}{3\pi-4}\right)$$

$$\theta = 47.88\ldots^\circ$$

$$\theta = 47.9^\circ \text{ (3sf)}$$

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Question 4 continued

(Total for Question 4 is 12 marks)

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5. A particle P moves in a straight line with simple harmonic motion about a fixed point O.
The magnitude of the greatest acceleration of P is $18\,\text{m s}^{-2}$

When P is 0.3 m from O, the speed of P is $2.4\,\text{m s}^{-1}$

The amplitude of the motion is a metres.

(a) Show that $a = 0.5$

(5)

(b) Find the greatest speed of P.

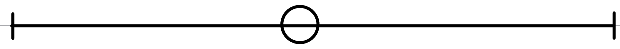
(2)

During one oscillation, the speed of P is at least $2\,\text{m s}^{-1}$ for S seconds.

(c) Find the value of S.

(4)

a)



Greatest acceleration occurs at zero displacement

$\ddot{x} = -\omega^2 x$ — MINUS sign represents acceleration is always directed towards O

where $\ddot{x}$ = acceleration

ω = angular speed

x = displacement/amplitude

ALSO: $v^2 = \omega^2(a^2 - x^2)$

FROM THIS

Using the above

$a\omega^2 = 18$

$\omega^2 = \dfrac{18}{a}$ ①

$2.4^2 = \omega^2(a^2 - 0.3^2)$ ②

V = velocity = $2.4\,\text{ms}^{-1}$
x = displacement = $0.3\,\text{m}$ — Info extracted from question

① into ②

$5.76 = \dfrac{18}{a}(a^2 - 0.3^2)$

$5.76 = 18a - \dfrac{1.62}{a}$

$0 = 18a^2 - 5.76a - 1.62$

Solving using calculator : $a = 0.5$

b) Solving for ω using ①

$\omega = \sqrt{\dfrac{18}{a}} = \sqrt{\dfrac{18}{0.5}} = 6 \text{ rad s}^{-1}$

Max speed : $V_{max} = a\omega$

$V_{max} = 0.5 \times 6 = 3\,\text{ms}^{-1}$

Question 5 continued

c) At $t=0$, $x=0$ so

starting at the centre of oscillation

$x = a \sin \omega t$

$\frac{dx}{dt} = v = a\omega \cos \omega t$

v is at least 2 ms^{-1} for S seconds

Substituting into expression above

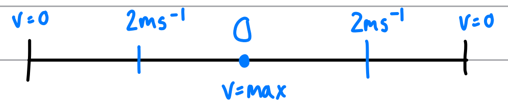
$2 = 0.5(6) \cos 6t$

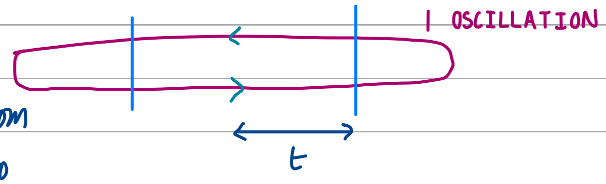
$2 = 3 \cos 6t$

$\frac{2}{3} = \cos 3t$

$t = \frac{1}{6} \arccos\left(\frac{2}{3}\right)$

$t = 0.140178\ldots$





time taken to decrease speed from max at centre of oscillation to $2ms^{-1}$

$\therefore$ Time where particle is travelling at least

$2 ms^{-1} = 4t = S$

$S = 4(0.140178\ldots) = 0.5607$ s

(Total for Question 5 is 11 marks)

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

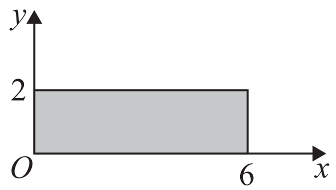


Figure 4

The shaded region, shown in Figure 4, is bounded by the x-axis, the line with equation $x = 6$, the line with equation $y = 2$ and the y-axis.

This region is rotated through 360° about the ***x*-axis** to form a solid of revolution. This solid is used to model a **non-uniform** cylinder of height 6 cm and radius 2 cm.

The mass per unit volume of the cylinder at the point (x, y, z) is $\lambda(x + 2)\,\text{kg cm}^{-3}$, where $0 \leqslant x \leqslant 6$ and λ is a constant.

(a) Show that the mass of the cylinder is $120\lambda\pi$ kg.

(3)

(b) Show that the centre of mass of the cylinder is 3.6 cm from O.

(4)

The point O is the centre of one end of the cylinder. The point A is the centre of the other end of the cylinder.

A uniform solid hemisphere of radius 3 cm has density $\lambda\,\text{kg cm}^{-3}$. The hemisphere is attached to the cylinder with the centre of its circular face in contact with the point A on the cylinder to form the model shown in Figure 5.

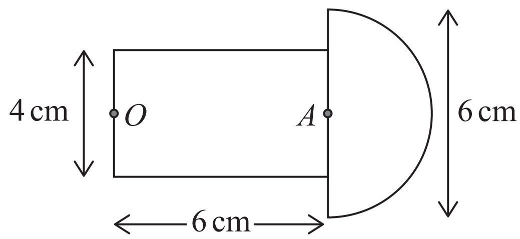


Figure 5

The model is placed with the end containing O on a rough inclined plane which is inclined at angle $\alpha°$ to the horizontal. The plane is sufficiently rough to prevent the model from sliding. The model is on the point of toppling.

(c) Find the value of α.

(6)

Question 6 continued

a) Mass $= \int_a^b \rho \pi y^2 \, dx$ [solid of revolution about x-axis]

$$\text{Mass} = \int_0^6 \lambda(x+2)\,\pi(2)^2\,dx$$

$$= 4\pi\lambda\left[\frac{x^2}{2} + 2x\right]_0^6$$

$$= 4\pi\lambda\left(\frac{36}{2} + 12\right)$$

Mass $= 120\lambda\pi$ kg

b) Taking moments about y-axis

$M\bar{x} = \int_a^b \rho \pi y^2 x \, dx$ [solid of revolution about x axis]

Substituting values into expression above

$$M\bar{x} = \int_0^6 \lambda(x+2)\,\pi(2)^2 x\,dx$$

$$= 4\pi\lambda \int_0^6 x^2 + 2x \, dx$$

$$= 4\pi\lambda\left[\frac{x^3}{3} + x^2\right]_0^6$$

$$= 4\pi\lambda\left(\frac{216}{3} + 36\right)$$

$M\bar{x} = 432\pi\lambda$

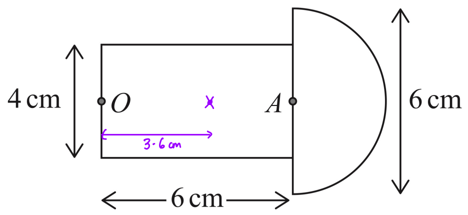
$$\bar{x} = \frac{432\pi\lambda}{120\pi\lambda} = \frac{432}{120} = 3.6 \text{ cm}$$

$\therefore$ COM of cylinder is 3.6 cm away

P 7 5 6 9 1 A 0 2 1 2 8

Question 6 continued

c)



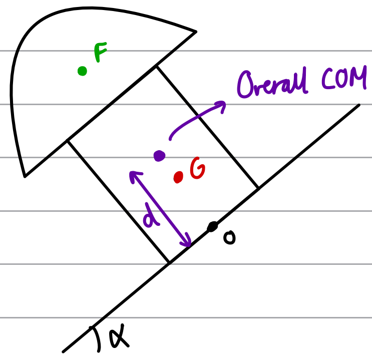
COM of solid hemisphere (in DF book)

$= \frac{3}{8} r$ from centre

$= \frac{3}{8} \times 3 = \frac{9}{8}$ cm

Volume of hemisphere $= \frac{4}{3}\pi r^3 \div 2$

$= \frac{4}{3}\pi (3)^3 \div 2 = 18\pi$



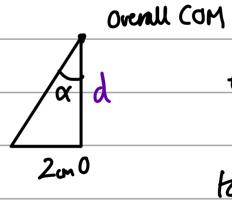
At point of toppling so overall COM lies directly on top of edge of base

Take moments about base diameter of base

CYLINDER HEMISPHERE

$120\lambda\pi \times 3\cdot6 + 18\lambda\pi \times \left(6 + \frac{9}{8}\right) = (120+18)\pi\lambda d$

$d = \frac{747}{184}$



$\tan\alpha = \frac{2}{d}$

$\tan\alpha = \frac{368}{747}$

$\alpha = \arctan\left(\frac{368}{747}\right)$

$\alpha = 26\cdot2°$

Question 6 continued

(Total for Question 6 is 13 marks)

7.

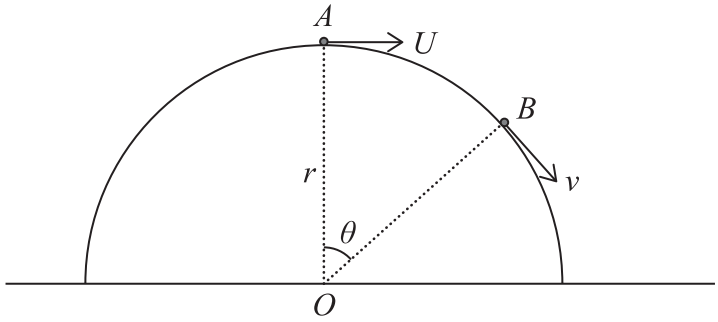


Figure 6

A smooth solid hemisphere has radius r and the centre of its plane face is O.
The hemisphere is fixed with its plane face in contact with horizontal ground, as shown in Figure 6.
A small stone is at the point A, the highest point on the surface of the hemisphere.
The stone is projected horizontally from A with speed U.
The stone is still in contact with the hemisphere at the point B, where OB makes an angle θ with the upward vertical.
The speed of the stone at the instant it reaches B is v.

The stone is modelled as a particle P and air resistance is modelled as being negligible.

(a) Use the model to find v^2 in terms of U, r, g and θ

(3)

When P leaves the surface of the hemisphere, the speed of P is W.

Given that $U = \sqrt{\dfrac{2rg}{3}}$

(b) show that $W^2 = \dfrac{8}{9}rg$

(5)

After leaving the surface of the hemisphere, P moves freely under gravity until it hits the ground.

(c) Find the speed of P as it hits the ground, giving your answer in terms of r and g.

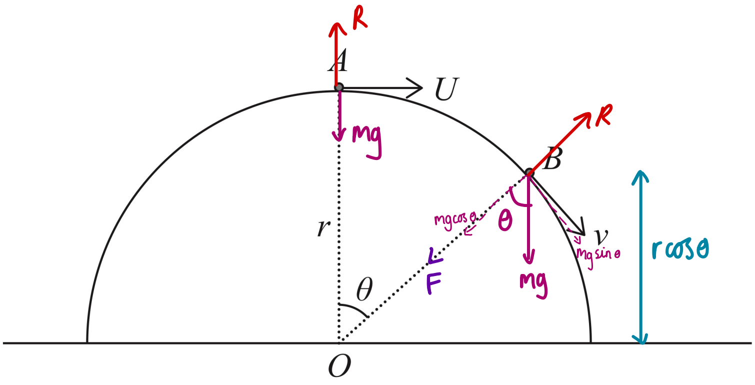
(3)

At the instant when P hits the ground it is travelling at $\alpha°$ to the horizontal.

(d) Find the value of α.

(3)

Question 7 continued



a) F = centripetal force

mg = weight

$F = \frac{mv^2}{r} = \frac{mW^2}{r}$

formulae memorised

Using conservation of energy equation from top position to new position

$\frac{1}{2}mU^2 + mgr = \frac{1}{2}mv^2 + mgr\cos\theta$

KE GPE KE GPE

Rearranging

$v^2 = U^2 + 2gr(1-\cos\theta)$

b) Equation for circular motion at B

$F = mg\cos\theta - R$

R = 0 when particle leaves surface and using formulae for F

$\frac{mW^2}{r} = mg\cos\theta$

$W^2 = rg\cos\theta$ ①

If $U = \sqrt{\frac{2rg}{3}}$

$v^2 = \frac{2rg}{3} + 2rg(1-\cos\theta)$

expression for W^2 ②

Setting ① = ②

$W^2 = \frac{8rg}{3} - 2rg\cos\theta$

$rg\cos\theta = \frac{8rg}{3} - 2rg\cos\theta$

$3rg\cos\theta = \frac{8rg}{3}$

$\cos\theta = \frac{8}{9}$ ③

Substituting ③ back into ①

$W^2 = \frac{8}{9}rg$

c) Using conservation of energy from take off to just before it hits the floor

$\frac{1}{2}mW^2 + mgr\cos\theta = \frac{1}{2}mV^2$

KE GPE

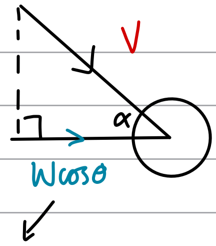
$W^2 + 2gr\cos\theta = V^2$

$V^2 = \frac{8}{9}rg + 2rg\cos\theta = \frac{8}{9}rg + 2rg\left(\frac{8}{9}\right) = \frac{8rg}{3}$

$V = \sqrt{\frac{8rg}{3}}\ \text{ms}^{-1}$

Question 7 continued

d)



$$\cos\alpha = \frac{W\cos\theta}{V}$$

$$= \frac{\sqrt{\frac{8}{9}rg}\left(\frac{8}{9}\right)}{\sqrt{\frac{8rg}{3}}} = \frac{8\sqrt{3}}{27}$$

Horizontal component is unchanged from when particle leaves surface

$$\alpha = \arccos\left(\frac{8\sqrt{3}}{27}\right)$$

$$= 59.122767\ldots$$

$$\alpha = 59.1^\circ \quad (3sf)$$

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Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS